

Intensity changes in the Doppler effect

(Lorentz invariant/gamma-ray bursts)

MONTGOMERY H. JOHNSON AND EDWARD TELLER

University of California, Lawrence Livermore National Laboratory, Livermore, California 94550

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ABSTRACT When a source moves in any direction, the source strength and the frequencies are altered by the Doppler effect. It is shown that the source strength divided by the cube of the frequency is a Lorentz invariant.

Ruderman (1) remarked that an emitting source moving toward the observer causes an intensity increase by a factor $(v/v')^4$, where v and v' are the received and emitted frequencies. In the following discussion, primed quantities refer to the source at rest and unprimed quantities to a source moving with velocity \vec{u} . For blackbody radiation at a temperature T' , this factor may be combined with $(T')^4$ to show that Stefan-Boltzman law gives the correct intensity when the temperature T of the moving source is taken to be $T'(v/v')$.

It is a common occurrence in optics that the diminishing intensity of a light source with increasing distance may be compensated by an increase in directionality. Indeed, the light can be refocused, and in that case the degree of disorder, corresponding to entropy, does not change. Ruderman's statement may be made in a way that generalizes the above statement to cases in which the Doppler effect shifts the frequencies. Then intensities must be compared for blackbody radiation temperatures that have been changed by the ratio v/v' .

The ratio of frequencies v/v' is designated u . We shall show that the ratio of the source strength for a source moving in any direction to the source strength of a stationary source contains two factors: the ratio of solid angles and the ratio of the quanta's emission rates. We shall see the first factor is u^2 and the second is u , so that the ratio of source strengths becomes u^3 . Here the source strength, I , is the energy radiated per unit frequency interval, per unit time, and per unit solid angle.

Let θ be the angle between the direction of emission and the source velocity \vec{u} , and let $\beta = |\vec{u}|/c$ (with c being the velocity of light). Because the magnitude of a quantum's momentum \vec{p} is $h\nu/c$ (with h being Planck's constant), we can write

$$P_{\parallel} = \gamma(P'_{\parallel} + \beta P') \text{ or } P \cos \theta = \gamma P' (\cos \theta' + \beta),$$

where

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Also

$$P_{\perp} = P'_{\perp} \text{ or } P \sin \theta = P' \sin \theta'.$$

Elimination of θ gives

$$\frac{P'}{P} = \frac{v'}{v} = \frac{1}{u},$$

where

$$u = \gamma(1 + \beta \cos \theta').$$

The ratio of shifted to unshifted frequency is $\gamma(1 + \beta \cos \theta')$.

We have seen that

$$\cos \theta = \frac{\gamma P'}{P} (\cos \theta' + \beta) = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'}$$

The solid angle into which radiation is emitted is $d\Omega = d(\cos \theta) d\phi$, where ϕ is the azimuth around \vec{u} . By taking the differential of the above equation and rearranging,

$$d \cos \theta = \frac{d \cos \theta'}{u^2}$$

Because $d\phi = d\phi'$, we have

$$d\Omega = d \cos \theta d\phi = \frac{d \cos \theta' d\phi'}{u^2} = \frac{d\Omega'}{u^2}$$

The solid angle into which the moving source radiates changes by the factor $1/u^2$ compared to the corresponding solid angle for the stationary source.

For the moving source, the wave number $k' = \frac{2\pi}{c}v'$ is changed by the factor u . Consequently, the density of quanta in a given wave train is multiplied by u , and the rate of emission into a given angular cone must be changed by the same factor. The rate at which quanta are emitted for the stationary source is $\frac{I' dv'}{h\nu'} d\Omega'$ and for the moving source is $\frac{I}{h\nu} dv d\Omega$. Hence,

$$\frac{I}{h\nu} dv d\Omega = u \frac{I'}{h\nu'} dv' d\Omega'.$$

Making use of the previous relations between v and v' and between $d\Omega$ and $d\Omega'$, we find $I(v, \theta, \phi) = u^3 I'(v', \theta', \phi')$. In short I/v^3 is a Lorentz invariant. Evidently the radiation integrated over the frequency spectrum is changed by the factor u^4 because each frequency is changed by the same factor u and the whole frequency range is expanded in the same ratio.

If the source has a uniform surface temperature T' , I' does not depend on the angles. Integration over the frequency spectrum yields a result proportional to $(T')^4$. The radiation from a source moving in any direction is correctly given by assigning a source temperature $T = uT'$. In an expanding or turbulent source, the observer sees the radiation surface elements move with different velocities. The above results must then be applied to each surface element before combining to give the radiation in a particular direction. When the velocities are relativistic, the radiation from elements moving toward the observer are strongly emphasized.

The above consideration, as in the case of Ruderman, may be usefully applied in discussing observed gamma-ray bursts of short duration.

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1. Ruderman, M. (1975) *Ann. N.Y. Acad. Sci.* **252**, 164.